Calculation of Magnetostriction Induced Deformations in Grain Oriented and Non-Oriented Silicon Iron Transformer Cores Thanks to an Imposed Magnetic Flux Method

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This work focuses on the development of an algorithm for the prediction of a transformer core deformation, using a magneto-mechanical approach. An imposed magnetic flux method coupled with finite elements is employed for the magnetic resolution. The constitutive law of the material uses a multi-scale model describing both magnetic and magnetostrictive anisotropies. Magnetostriction is introduced as an input free strain of a mechanical problem to get the deformation and displacement fields. The numerical process is applied to compare the deformations of a given magnetic circuit made of Grain Oriented and Non-Oriented FeSi.

Index Terms—**Magnetostriction, transformers, multiscale modeling, iron-silicon alloys, finite element method.**

I. Introduction - Objectives

THE ELECTRICAL power of aircraft on-board system is increasing with the proliferation of comfort equipments and the gradual replacement of hydraulic actuators with electrical actuators. As a consequence higher power transformers are needed to accord the voltage to the different electrical devices. Magnetic materials are usually dense, which costs a lot of the board weight. One way to reduce the mass is to use higher power density materials as iron-cobalt materials. Transformers that use these materials generate unfortunately a loud noise in operation caused by periodic deformations of sheets of the transformer core, that strongly limits the use of these materials for this application. This deformation has two origins: i) elastic strain associated to magnetic forces appearing on the free surface and volume; ii) spontaneous magnetostriction depending on the local magnetic state of the material. In the application considered in this paper, the contribution of magnetic forces is negligible. One interesting solution to reduce the macroscopic magnetostrictive deformation is to develop crystallographic textures to orient the magnetic domains distribution. This behavior has been observed for example, with grain-oriented materials [1]. Optimization of such a transformer requires to use a numerical model where magnetostriction strain is an output of the constitutive behavior of the material.

II. Global modeling strategy

A. Constitutive law

The constitutive law used for the modeling is a simplified version of a full multi-scale magneto-mechanical model (MSM) [1], [2]. In the complete version, the considered scales are the magnetic domain, the single crystal and the polycrystalline (macroscopic) scales. This model allows an accurate modeling of anhysteretic magnetic and magnetostrictive behaviors of ferro/ferrimagnetic materials, and takes the effect a mechanical stress into account. The number of internal variables of such model is nevertheless too high to be implemented in a complex structure model with a high degrees of freedom number. The simplified version (SMSM) where the magnetic material is considered as an equivalent single-crystal (including anisotropy effects) has been recently proposed for that purpose [3]. This single-cristal consisting of magnetic domains is oriented to different directions. Local free energy of a magnetic domain (α) oriented in direction ($\vec{\gamma}_\alpha$) is expressed as the sum of three contributions if the stress effect is neglected (weak coupling):

$$W^\alpha_{tot} = W^\alpha_{mag} + W^\alpha_{an} + W^\alpha_{conf} \tag{1}$$

W^α_{mag} is the Zeeman energy, introducing the effect of the applied magnetic field on the equilibrium state. W^α_{an} is an anisotropic energy term related to the crystallographic texture and the magneto crystalline anisotropy. W^α_{conf} is a configuration energy term, which allows some peculiar initial distribution of domains (residual stress effect, demagnetizing surface effect...). A Boltzmann like function is used to calculate the volume fraction of domains in different directions f_α.

$$f_\alpha = \frac{exp\left(-A_s W^\alpha_{tot}\right)}{\sum_\alpha exp\left(-A_s W^\alpha_{tot}\right)} \tag{2}$$

Where A_s is an adjusting parameter. Macroscopic quantities are finally obtained by averaging the microscopic quantities over the single crystal volume (3)(4).

$$\vec{M} = \sum_\alpha f_\alpha \vec{M}^\alpha \tag{3}$$

$$\boldsymbol{\epsilon}_\mu = \sum_\alpha f_\alpha \boldsymbol{\epsilon}^\alpha_\mu \tag{4}$$

From a given magnetic field $\vec{H}$, the simplified MSM gives then the corresponding free magnetostriction strain and magnetization.

B. Magnetic resolution with imposed magnetic flux method

One important criterion for power transformer design is power-to-mass ratio (transmitted power per unit mass), which is proportional to the magnetic flux ϕ circulating in the transformer core. Imposing the same magnetic flux is thus interesting to compare the relevance of different materials for a same structure. The partially coupled problem is solved using a sequential approach: magnetic resolution at a given flux first (leading to the local magnetostriction), mechanical resolution in a second step. The magnetic resolution is using an iterative fixed-point method [4]: a magnetic flux ϕ is imposed; the magnetization $\vec{M}$ is arbitrary defined at the first loop allowing a first estimation of the magnetic field $\vec{H}$. The magnetization is then updated, using the SMSM. The procedure is iterated until convergence. Combined with basic Maxwell equation $div(\vec{B}) = 0$, the re-written constitutive equation in weak formulation is shown in (5). A second equation (6) is obtained from the formulation of total magnetic energy $\phi I = \int_\Omega \vec{T} \cdot \vec{B} \, \mathrm{d}\Omega$, with current potential vector $\vec{T}$ and magnetic induction $\vec{B}$ [5]. Ω is the integration domain and υ is a test function. A unit current potential vector $\vec{T_0}$ is imposed to get $I \vec{\nabla} \times \vec{T_0} = \vec{j}$. The non-linear problem is solved at a given applied flux leading to current value $\vec{j}$ in the coils (hence magnetic field).

$$\int_\Omega \mu_0 \vec{\nabla}\omega \cdot \vec{\nabla}\upsilon \, \mathrm{d}\Omega + I \int_\Omega \mu_0 \vec{T_0} \cdot \vec{\nabla}\upsilon \, \mathrm{d}\Omega = - \int_\Omega \mu_0 \vec{M} \cdot \vec{\nabla}\upsilon \, \mathrm{d}\Omega \tag{5}$$

$$\int_\Omega \mu_0 \vec{\nabla}\omega \cdot \vec{T_0} \, \mathrm{d}\Omega + I \int_\Omega \mu_0 \vec{T_0} \cdot \vec{T_0} \, \mathrm{d}\Omega = \phi - \int_\Omega \mu_0 \vec{M} \cdot \vec{T_0} \, \mathrm{d}\Omega \tag{6}$$

C. Mechanical resolution

The free magnetostrictive strain $\boldsymbol{\epsilon}_\mu$ calculated from the SMSM is then transformed into an equivalent force density $\vec{f_{eq}}$ as a body force of a classical plane stress mechanical problem. This equivalent force density is calculated from $\boldsymbol{\epsilon}_\mu$ thanks to: $\vec{f_{eq}} = -\vec{\nabla} \cdot (\mathbb{C} : \boldsymbol{\epsilon}_\mu)$, where $\mathbb{C}$ is the stiffness tensor of the medium. Mechanical resolution is carried out for each harmonic component of this equivalent force density using a Fast Fourier Transformation (FFT) method. Inverse FFT after resolution leads to the deformation at each node of the transformer core over the time.

III. Deformation of a '8'-shape transformer core made of Grain Oriented and Non-Oriented FeSi

We propose here to compare the relevance of Grain Oriented (GO) or perfectly Non-Oriented (NO) FeSi for an ideal '8'-shape structure (no air-gap). GO FeSi is processed in such a way that it offers better magnetic properties than NO FeSi in the rolling direction (RD), due to a dominant quantity of domains oriented along this direction [1]. Fig. 1 allows to compare the magnetic and magnetostrictive behaviors in the direction of applied field for both materials (Parameters of the SMSM for both materials will be given in the full paper), illustrating the strong anisotropy of GO material, and

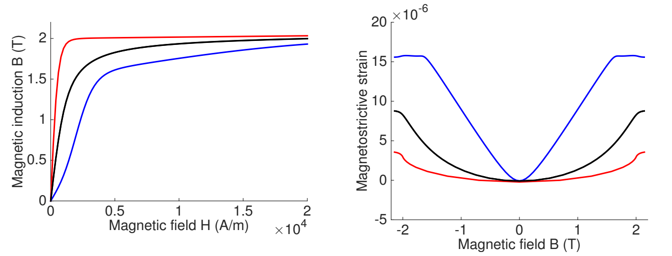

Fig. 1. Simplified multi-scale modelling : anhysteretic magnetization curves (left) and longitudinal magnetostriction (right) for isotropic NO FeSi (black line) and GO FeSi (red: $\vec{H}//RD$; blue: $\vec{H}//TD$ - *Transverse Direction*).

especially a very low magnetostriction amplitude along RD (in accordance with experimental results [1]). For simplification reasons, the three-phase power transformer is excited only by a central coil imposing a sinusoidal flux which leads to a maximum induction of $B = 1.5$T. Amplitude of the exciting current and core deformation are then solved. Some results are shown in Fig.2 (with RD in the direction Y).

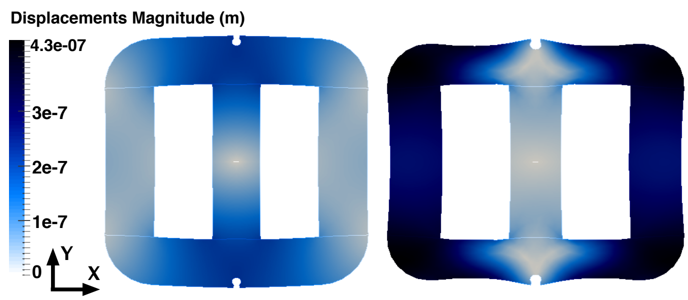

Fig. 2. Magnitude of displacement field of transformer core: NO FeSi (left); GO FeSi (right), $B = 1.5$T.

The magnetic conditions lead to an exciting current of 421 Ampere-turns for NO FeSi and of 355 Ampere-turns for the GO FeSi. However NO FeSi core generates less displacement than GO FeSi because a significant part of magnetic flux is directed along TD ($\vec{H}//X$) that leads to a high magnetostriction for GO material. Therefore, an geometric optimization would be necessary to take advantage of the GO FeSi by increasing the section area where the magnetic field is aligned with TD (reducing the magnetization and magnetostriction levels). A new structure and associated simulation results will be given in the full paper.

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